SELF-REFRACTION OF IMPULSE AND QUASI-STATIONARY ACOUSTIC BEAMS CONTAINING SHOCK FRONTS IN A NEWTONIAN FLUID

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Abstract: The self-refraction of some beams in a Newtonian fluid is theoretically studied. A single pulse that takes the shape of an isosceles triangle at a transducer and a solitary shock wave are considered as examples. The novelty lies in the consideration of a quasi-stationary shock wave (with positive or negative pressure) and negative triangular pulses. We conclude that waveforms with negative excess pressure undergo self-focusing, in contrast to those with positive excess pressure. The difference in self-refraction of quasi-stationary and impulse beams is revealed by means of numerical modeling.

Keywords: self-refraction of acoustic beam, self-focusing of sound, self-action of beams with shock fronts

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1. Introduction

The waveforms with shock fronts are of great importance not only in the theory of nonlinear acoustics, but also in technical and medical applications of ultrasound. A single acoustic pulse in a weakly damping fluid acquires usually the shape of N-wave. There exist stationary planar waveforms with the shock fronts which propagate in Newtonian fluids due to joint action of nonlinearity and absorption. Smooth shock fronts form also in highly viscous tissues. Their speed may differ from the linear sound speed in a medium, if absolute value of the negative peak pressure differs from the positive peak pressure at a shock. The only parameters of these steady waveforms are the peak values of acoustic pressure before and behind the shock which determine also a width of a shock. The shock waves are self-similar, their shape does not vary in the course of propagation. These waveforms are quasi-stationary in slightly diverging flows. Since any initial perturbation develops into shock waves in a weakly damping flow, nonlinear effects associated with the propagation of waveforms with shocks are of especial importance.

If the maximum acoustic pressure in a planar shock wave exceeds the absolute minimum value, the shock front propagates with a speed higher than the linear sound speed in the medium, and vice versa [1]. This happens due to the nonlinear distortions of a signal of a finite magnitude. The reason for self-refraction in acoustic beams is the variation of the shock front excess speed Δc in different parts of the beam cross-section due to a variation in the magnitude of the excess pressure with a distance from the beam axis [2, 3]. A nonlinear excess speed Δc of the shock front speed grows with the pressure step A [1]. As the value of A in the vicinity of the beam axis is larger than in remote regions, self-refraction and flattening of the focused wave front are observed. The nonlinear absorption, which is stronger in the par-axial area, makes the transverse distribution of A even more uniform and results in an almost flat wave front in the paraxial area near the focus. Evidently, the nonlinear focus shifts with respect to the geometrical one (farther from the transducer) and the beam waist enlarges. The self-refraction was observed in the experiments described in the review [4]. The theoretical justification and evaluations of self-defocusing of an impulse sound which propagates in an inviscid fluid before and after formation of a discontinuity, may be found also in [3, 5, 6]. In these publications, positive triangular impulses at the transducer were considered. In the case of an impulse in which the sum of positive and negative peak acoustic pressures in discontinuity is negative, one may expect that self-refraction leads to additional focusing of the beam and displaces the focus towards the transducer. Impulses with negative pressure jump have not been discussed in previous publications. The shock wave with a negative acoustic pressure also should experience focusing, if the sum of positive and negative peak acoustic pressures in a shock is negative. In this study we consider the self-focusing and self-defocusing of impulsive and shock beams with positive and negative acoustic pressures.

As for a stationary shock wave, it is well-known that it cannot exist in a purely nonlinear flow due to nonlinear distortions. While an impulse expands and reduces its peak pressure during propagation in a pure nonlinear flow, a single planar shock wave may form only due to the joint impact of nonlinearity and attenuation. The symmetric solitary wave which propagates with the linear sound speed does not experience self-refraction, since it possesses equal absolute values of maximum and minimum acoustic pressure, and there are no variations in speed of a shock front with a magnitude of a jump and with the distance from the beam axis. The symmetric shock wave propagates with a linear sound velocity. The equations describing the self-action of a solitary shock wave beam in a Newtonian fluid are fairly complex to an analytical solution, if the shock is broad. They may be essentially simplified in the case of a waveform with a very short shock front, i.e., in the case of comparatively weak attenuation. Self-refraction due to variations in the shock front speed differs from the thermal self-action of a beam which occurs due to nonlinear acoustic heating of a medium with attenuation. Acoustic heating may be caused by pure nonlinear dissipation at the the shock wave discontinuity. Thermal self-action is observed in the case of beams with and without discontinuities, for any kind of acoustic exciters. It represents heating of a medium due to losses in the acoustic energy which in turn affects the sound speed. It occurs in planar flows as well, but in the case of beams, it may change the focal distance due to uneven heating in layers perpendicular to the beam axis. The example of thermal self-action of periodic saw-tooth impulses was firstly considered by Rudenko and co-authors in [6]. Both self-refraction and thermal self-action of sound are nonlinear phenomena.

2. Beams focused at a transducer

An equation which describes the evolution of the acoustic pressure p in a slightly divergent symmetric beam which propagates in the positive direction of axis Ox in a Newtonian fluid, takes the form [7]:

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial t}+c_0\frac{\partial p}{\partial x}+\frac{\varepsilon}{c_0\rho_0}p\frac{\partial p}{\partial x}-\frac{b}{2c_0^2\rho_0}\frac{\partial^2 p}{\partial x^2}\right)=\frac{c_0}{2}\Delta_\perp p, \tag{1}$$

This is one of the forms of the celebrated Kuznetsov-Zabolotskaya-Kuznetsov equation (KZK) [7–9] which describes the dynamics of a slightly divergent acoustic beam. Eq. (1) accounts neither for the thermal nor inertial self-action of a beam in a fluid; x and r are cylindrical coordinates, c_0 denotes an infinitely small-signal sound speed in a fluid, and ρ_0 denotes unperturbed density, t designates time, $\Delta_\perp$ is the Laplacian with respect to the radial coordinate r, ε is the parameter of nonlinearity of a fluid, and b designates the total attenuation due do mechanical and thermal losses.

We start from establishing the stationary shock wave in a planar flow, i.e., a stationary solution of the equation

$$\frac{\partial p}{\partial t}+c_0\frac{\partial p}{\partial x}+\frac{\varepsilon}{c_0\rho_0}p\frac{\partial p}{\partial x}-\frac{b}{2c_0^2\rho_0}\frac{\partial^2 p}{\partial x^2}=0. \tag{2}$$

Let us find acoustic pressure as a function of $\tau=t-x/c$ and μx, where μ is a small parameter. It is responsible for slow variations of the waveform in the course of propagation due to nonlinear distortion and attenuation. Collecting the leading-order terms of order μ^0 (completely reduced) and μ^1 rearranges the Burgers equation, Eq. (2) into the form

$$\frac{\partial p}{\partial x}+\left(\frac{1}{c_0}-\frac{1}{c}\right)\frac{\partial p}{\partial \tau}-\frac{\varepsilon}{c_0^2 c\rho_0}p\frac{\partial p}{\partial \tau}-\frac{b}{2c_0c^2\rho_0}\frac{\partial^2 p}{\partial \tau^2}=0. \tag{3}$$

We make use of the method of different scales applied in many problems in acoustics, see Refs [1, 7, 8]. The stationary solution of Eq. (3) (i.e., the solution which depends exclusively on τ and does not depend on the distance from the exciter), takes the form

$$p=\frac{A}{2}\left(1+\tanh\left(\frac{Ac\varepsilon\tau}{2bc_0}+\frac{\ln(\varepsilon)}{2}\right)\right)\approx\frac{A}{2}\left(1+\tanh\left(\frac{A\varepsilon\tau}{2b}+\frac{\ln(\varepsilon)}{2}\right)\right) \qquad (4)$$

and propagates in the positive direction of axis Ox with speed c which exceeds the linear sound speed c_0, if $A>0$, or is smaller, if $A<0$:

$$c=c_0+\frac{A\varepsilon}{2c_0\rho_0}. \qquad (5)$$

We consider a small difference between c and c_0, since

$$\frac{|c-c_0|}{c_0}=\frac{|A|\varepsilon}{2c_0^2\rho_0}=\frac{\varepsilon}{2}M, \qquad (6)$$

where M is the Mach number which is small in weakly nonlinear flows. An acoustic pressure in the considered planar solitary wave tends to zero when τ tends to minus infinity ($A>0$), or when τ tends to plus infinity ($A<0$). Hence, an excess pressure in the shock wave may be of two kinds, positive or negative.

Hence, an excess pressure in the shock wave may be of two kinds, positive or negative. In both cases, the jump of pressure in the stationary impulse equals $|A|$. The duration of the shock wave depends on the jump of magnitude at the shock front, it equals $2b/|A|\varepsilon$. We will make use of the stationary waveform as the leading-order solution to Equation (1) in studies of self-refraction of weakly diverging beams.

Recalling the theory of geometrical acoustics,we consider a slightly divergent beam introducing a new variable θ, which is referred to the eikonal $\psi(x,r)$ (i.e., additive in the phase which depends in general on x and r):

$$\theta=t-x/c_0-\psi(x,r)/c_0. \qquad (7)$$

We consider p as a function of x, r, θ. We treat A as a function of x and r, and consider a limit when the width of a shock front tends to zero. This is a limit of comparatively small damping, i.e., large Reynolds numbers of a flow. The leading-order dynamic equations describing an acoustic pressure in a slightly divergent beam, take the form

$$\frac{\partial p}{\partial x}-\frac{\varepsilon}{c_0^3\rho_0}\left(p-\frac{A}{2}\right)\frac{\partial p}{\partial\theta}+\frac{\partial\psi}{\partial r}\frac{\partial p}{\partial r}+\frac{\Delta_\perp\psi}{2}p=0, \qquad (8)$$

$$\frac{\partial\psi}{\partial x}+\frac{1}{2}\left(\frac{\partial\psi}{\partial r}\right)^2=-\frac{\varepsilon A}{2c_0^2\rho_0} \qquad (9)$$

They follow from substitution of $p(x,r,\theta)$ to Equation (1). Equations (8) and (9) account for a nonlinear variation $\Delta c=c-c_0$ in the shock velocity which increases with the enlargement of the pressure step $|A|$. Since the values of $A(x,r)$ and c differ at the beam axis and on its periphery, the wave front varies in the course of propagation. The system of Equations (8) and (9) is similar with the one which describes the peak pressure A of a single pulse [3] (see also Equations (25) and (26) from [5]). We have derived it for a wave with a narrow shock front in a Newtonian fluid, but it may be obtained directly supposing the pure nonlinear attenuation of a solitary shock wave with discontinuity. Equations (8) and (9) are valid in these cases where the acoustic nonlinearity is important, and diffraction and damping are comparatively weak, i.e., in the approximation of geometrical acoustics. Rearranging the variables and introducing an unknown function $f(x)$ which describes variations in the beam width and in the peak pressure,

$$P = f(x)p, \quad B = f(x)A, \quad \zeta = \frac{r}{af}, \quad \xi = \int_0^x \frac{dx'}{f(x')}, \tag{10}$$

where a denotes the initial beam width, the transport Equation (8) is readily reduced to the equation recalling that for a simple wave,

$$\frac{\partial P}{\partial \xi} - \frac{\varepsilon}{c_0^3 \rho_0} \left(P - \frac{B}{2} \right) \frac{\partial P}{\partial \theta} = 0. \tag{11}$$

A parabolic wave front in the general form is also assumed [3, 5]:

$$\psi(x,r,t) = \phi(x,t) + \frac{r^2}{2} \frac{\partial}{\partial x} \ln f(x,t). \tag{12}$$

This means that the eikonal at any x, t supports the waveform sphericity. It is its curvature only that may vary in the course of the beam propagation. The unknown function $f(x)$ is responsible for these variations. The solution to Equation (11) with an account for (12), is represented by the formula

$$A(x,r) = \frac{P_0}{f} \Phi\left(\frac{r}{af} \right), \tag{13}$$

where function Φ reflects the transverse distribution of the acoustic pressure at the transducer, and P_0 designates the initial peak acoustic pressure at the transducer and at the beam axis. Eq. (13) differs from Eq. (29) from [5] by the last term in parentheses which is absent in Eq. (13). This term formally tends to zero as the shock formation distance in a planar triangular impulse x_s tends to infinity, where x_s is the distance at which a break is formed in the planar triangular impulse of initial duration $2T_0$ and amplitude P_0,

$$x_s = \frac{c_0^3 \rho_0 T_0}{\varepsilon P_0}. \tag{14}$$

We consider a Gaussian beam with $\Phi(\xi) = \exp(-\xi^2)$. Expanding (13) in the vicinity of the beam propagation axis and substituting it in the eikonal equation (9), we arrive at the equation for the unknown function $f(x)$ in the case of the solitary shock wave,

$$f^2 \left(\frac{d^2 f}{dx^2} \right) = \widetilde{\Pi}, \tag{15}$$

where

$$\widetilde{\Pi}=\frac{\varepsilon P_0}{a^2c_0^2\rho_0}. \tag{16}$$

Eq. (15) should be solved with the boundary conditions for a focused beam at the transducer (situated at $x=0$) with the initial curvature R^{-1}:

$$f=1, \quad \frac{df}{dx}=-\frac{1}{R}. \tag{17}$$

To compare the features of propagation of the solitary shock front with initially triangular impulse, we make use of Eq. (30) from [5]. The following equation describes the dynamics of a single pulse which takes the shape of an isosceles triangle at the transducer with the duration $2T_0$:

$$f^2\left(\frac{d^2f}{dx^2}\right)=\widetilde{\Pi}\left(1+\frac{1}{4x_s}\int\limits_{x_1}^{x}\frac{dx'}{f(x')}\right)\left(1+\frac{1}{2x_s}\int\limits_{x_1}^{x}\frac{dx'}{f(x')}\right)^{-3/2}, \tag{18}$$

where

$$x_1=R(1-\exp(-x_s/R)) \tag{19}$$

is the distance at which a break is formed in the focused wave which has initially curvature R^{-1}. Actually, the distances after the shock formation are of interest, $x>x_1$. At the distances closer to the transducer, f equals simply $1-x/R$, and the wave front moves with the linear sound speed. The solution (13) admits negative A, i.e., negative peak pressure. Musatov, Rudenko et. al [3, 5] did not consider self-focusing of a negative impulse, though the formula derived by them for initially triangular impulse, Eq. (18), describes also an impulse with negative peak pressure. This matches negative P_0 and $\widetilde{\Pi}$. The boundary conditions for Eq. (18) at $x=x_1$, sound

$$f=1-\frac{x_1}{R}, \quad \frac{df}{dx}=-\frac{1}{R}. \tag{20}$$

The Equations (15) and (18) may be readily rearranged in the non-dimensional co-ordinate

$$z=\frac{x}{R}$$

as

$$f^2\left(\frac{d^2f}{dz^2}\right)=\frac{\Pi D}{2} \tag{21}$$

for solitary wave, and

$$f^2\left(\frac{d^2f}{dz^2}\right)=\frac{\Pi D}{2}\left(1+\frac{\Pi}{4}\int\limits_{1-\exp(-1/\Pi)}^{z}\frac{dz'}{f(z')}\right)\left(1+\frac{\Pi}{2}\int\limits_{1-\exp(-1/\Pi)}^{z}\frac{dz'}{f(z')}\right)^{-3/2} \tag{22}$$

for isosceles triangular impulse with

$$\Pi = \frac{R}{x_s}, \quad D = \frac{R}{x_d}, \tag{23}$$

where

$$x_d = \frac{a^2}{2c_0 T_0} \tag{24}$$

is the typical diffraction scale of an impulse. The second formula from Equations (21) and (22) describes the dynamics of an impulse with discontinuity at $z \geq 1 - \exp(-1/\Pi)$. It is reproduced from Ref. [3]. If $z < 1 - \exp(-1/\Pi)$, a shock front has not formed yet, and an impulse beam converges to a geometrical focus. An acoustic dimensionless pressure at the beam axis takes the form for the solitary shock wave of:

$$\frac{A(z,0)}{P_0} = \frac{1}{f}, \tag{25}$$

and of

$$\frac{A(z,0)}{P_0} = \frac{1}{f\left(1 + \frac{\Pi}{2}\int_{1-\exp(-1/\Pi)}^{z} \frac{dz'}{f(z')}\right)^{1/2}}. \tag{26}$$

for an impulse beam.

Fig. 1 shows the distribution of a dimensionless magnitude at the beam axis, $\frac{A(z,0)}{P_0}$, for a positive solitary shock wave and a single triangular pulse. The curves corresponding to triangular at the transducer impulse were re-calculated numerically due to the equation reported in Ref. [3] (the second equation from the set (21)-(22) with the boundary conditions (20)). All evaluations were made with use of *M*athematica.

All plots refer to the weak diffraction determined by small $D = 0.01$ and to different Π. While at comparatively small initial magnitudes of pressure the geometrical and nonlinear focuses nearly overlap (case $\Pi = 5$), the focal distances behave differently in the case of stronger nonlinearity. The focus for solitary shock shifts towards a transducer, and the focus for the triangular impulse shifts apart from a transducer. The peak values in the case of a shock solitary wave exceed the values of the triangular impulse for large Π. Some sagging in the peak amplitude in the vicinity of the transducer which specifies propagation of a triangular impulse is absent in the case of the shock solitary wave dynamics.

3. Beams planar at the transducer

As for the monopolar positive impulses, they always undergo defocusing due to nonlinear self-refraction because of the acceleration of the paraxial region of the shock front. A positive solitary shock wave also undergoes defocusing. However, a negative monopolar impulse and a negative solitary shock wave beam are self-focusing. In order to avoid discussions of the effects which are connected with the initial curvature of the beam front, let us consider planar beams at the transducer. It is pertinent to rearrange the equations governing function f which

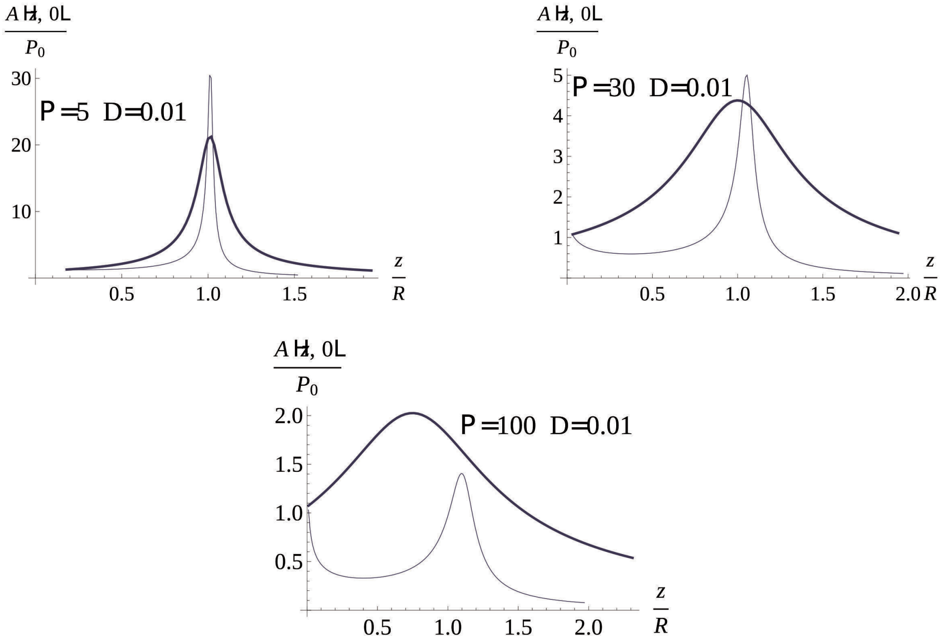

Figure 1. Dimensionless magnitude of acoustic pressure at the beam axis, $A(z, r=0)/P_0$, as a function of the dimensionless distance from the transducer, z/R. The bold lines relate to the solutions of the first equation from the set Eq. (21)-(22) for a solitary positive shock, and the thin lines relate to the solutions of the second equation from the set Eq. (21)-(22) for a single positive impulse, which is triangular at the transducer, for different $\Pi = R/x_s$ and $D = R/x_d = 0.01$

refers to a solitary shock wave or to an impulse, Eqs. (15) and (18), in the new variable z, read:

$$f^2\left(\frac{d^2f}{dz^2}\right) = \frac{\Pi}{2}, \tag{27}$$

for solitary wave and

$$f^2\left(\frac{d^2f}{dz^2}\right) = \frac{\Pi}{2}\left(1+\frac{1}{4}\int_1^z \frac{dz'}{f(z')}\right)\left(1+\frac{1}{2}\int_1^z \frac{dz'}{f(z')}\right)^{-3/2}, \tag{28}$$

for isosceles triangular impulse, where

$$\Pi = \pm\frac{x_s}{x_d}, \quad z = \frac{x}{x_s}. \tag{29}$$

Positive Π corresponds to waveforms with positive acoustic pressure, and negative Π corresponds to waveforms with negative acoustic pressure. The boundary conditions at $z=0$ for the solitary shock wave, and at $z=1$ for a triangular impulse The boundary conditions at $z=0$ for the solitary shock wave, and at

$z=1$ for a triangular impulse (i.e., at distances where discontinuity has already formed), take the form of

$$f=1, \quad \frac{df}{dz}=0. \tag{30}$$

An acoustic dimensionless pressure at the beam axis for an isosceles triangular impulse is

$$\frac{A(z,0)}{P_0}=\frac{1}{f\left(1+\frac{1}{2}\int_1^z \frac{dz'}{f(z')}\right)^{1/2}}, \tag{31}$$

and that for a solitary shock wave is described by the Equation (25). In the case of an initially planar wave, f depends on only one parameter Π. Fig. 2 shows the dimensionless magnitude of acoustic pressure at the axis of a positive or negative solitary shock wave beam, $A(z,r=0)/P_0$, which is planar at the transducer.

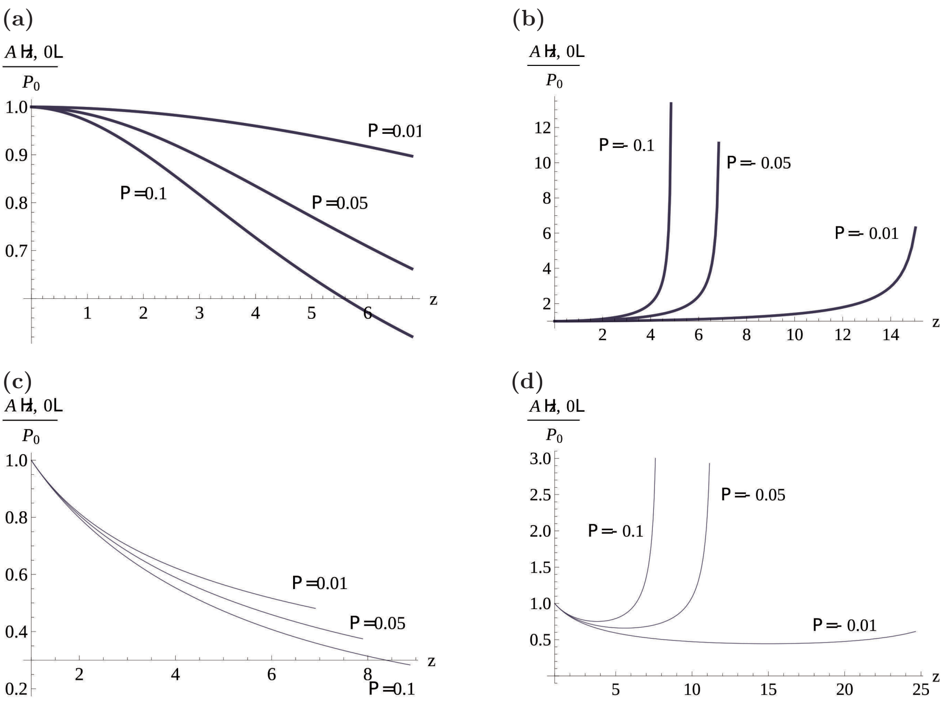

Figure 2. Dimensionless magnitude of acoustic pressure at the beam axis, $A(z,r=0)/P_0$ in a solitary shock wave (a,b) and in a monopolar impulse (c,d) (established by the first and second equations from the set Equation (27)-(28)) at different $\Pi=\pm x_s/x_d$. Positive Π corresponds to waveforms with positive acoustic pressure, and negative Π corresponds to waveforms with negative acoustic pressure. $z=x/x_s$ designates the dimensionless distance from the transducer

In the case of positive Π, the peak pressure decreases and the beam is evidently self-defocusing, while in the case of negative Π, it is self-focusing. Self-focusing is much more pronounced in the case of a single solitary shock wave, but self-defocusing is larger in the case of an initially triangular impulse of the same pressure step. The solitary shock wave does not reduce its magnitude in the vicinity of the transducer due to nonlinearity in contrast to a triangular impulse. Some sagging which is specific for the focusing triangular impulse, also takes place in the vicinity of a planar exciter. It is absent in the case of the dynamics of a shock solitary wave. In these examples, diffraction is small, i.e., the conclusions are valid if $|\Pi| \ll 1$.

4. Conclusions

This study considers self-refraction of some focused waveforms or waveforms with shock fronts planar at the transducer. The new conclusions refer to a single negative triangular initially monopolar impulse and a positive or negative shock stationary wave and to a comparative analysis of self-refraction. The previous results concerned the dynamics of initially triangular positive impulses. The peak pressure of initially focused positive beams enlarges while approaching the focus and reaches its maximum at some point. As compared with a single initially triangular positive pulse (whose self-refraction has been analyzed in details [3]), the focused solitary shock wave may form a nonlinear focus closer to the transducer, if nonlinearity is strong. In the case of an impulse, this point is located somewhat behind the geometrical focus $x = R$. Behind the nonlinear focus, the peak pressure decreases due to both the geometrical divergence and the nonlinear absorption. It has been well-established that self-refraction (i.e. self-defocusing) of positive impulses reduces additionally the maximum peak pressure in an impulse as compared to its pure nonlinear attenuation in one-dimensional geometry [3, 10]. As for the solitary shock wave, its magnitude achieves much smaller values in the case of weak nonlinearity but relatively enlarges in the case of weak nonlinearity as compared to the triangular impulse of the same magnitude at the transducer. This reflects the fact that the planar solitary wave propagates without variations of its magnitude as opposed to an impulse. An impulse undergoes nonlinear distortions until formation of discontinuity. At larger distances, its peak pressure rapidly decreases due to nonlinear attenuation at the shock front. In contrast, the solitary planar shock front is stationary due to the joint impact of nonlinearity and Newtonian attenuation, even if the shock front width is fairly small due to comparatively weak damping. In turn, during the propagation of an initially triangular impulse beam, after formation of discontinuity, nonlinear absorption starts to dominate and, despite the focusing, the peak pressure can even decrease in the vicinity of the transducer. This does not happen to a solitary shock wave, in all probability due to weak nonlinear distortions of a quasi-stationary waveform which is steady.

The conclusions are valid in the case of a solitary shock beam which propagates in a weakly attenuating Newtonian fluid with a steep and narrow shock front. That imposes relatively small Newtonian attenuation as compared with nonlinearity. Solitary shock waves which propagate with speeds different from the linear sound speed c_0, were considered. It is only these waveforms that experience self-refraction. The self-refraction is not observed during the propagation of a symmetric solitary shock wave beam the maximum pressure of which equals the absolute value of minimum pressure. Independently of the pressure step, the front of this symmetric wave propagates with the constant speed c_0. Hence, the wave front at the periphery and at the beam axis is not distorted as the beam propagates. The self-refraction of a solitary shock wave with zero pressure at $\tau \to \pm\infty$ enlarges with the pressure step $|A|$, i.e., with the difference of the shock wave speed and the linear sound speed, $|A|\varepsilon/2c_0\rho_0$.

The self-refraction of a solitary shock wave with zero pressure at $\tau \to \pm\infty$ enlarges with the pressure step $|A|$, i.e., with the difference of the shock wave speed and the linear sound speed, $|A|\varepsilon/2c_0\rho_0$.

In this study, the self-refraction of waveforms with negative acoustic pressure is firstly considered. Negative waveforms behave differently than positive ones: they are self-focusing due to refraction. There is an evident difference in self-refraction of a negative solitary shock wave and a triangular impulse. As for initially planar waveforms, the pressure step in a solitary shock wave of the same magnitude enlarges with distance much faster. The nonlinear attenuation makes the peak pressure of a triangular impulse to decrease at the vicinity of a transducer. That does not happen to a solitary shock wave beam. The pure nonlinear attenuation at the discontinuity yields the effects of self-focusing or self-defocusing proportional to A^3, while they are proportional to A in the self-refraction due to difference of the shock speed and the speed of infinitely-small magnitude sound proportional to A. Hence, the nonlinear variation of the local sound speed is of major importance. Self-refraction of acoustic beams with discontinuities differs substantially from their thermal self-action, which is determined by the temperature coefficient $\delta = c^{-1}\left(\partial c/\partial T\right)_p$ [6, 5]. Acoustic heating is proportional to A^2. Both self-refraction and thermal self-action are nonlinear phenomena and yield variations in the beam profile in the course of propagation, but self-refraction is more pronounced.

The results may be useful in remote studies of beam propagation media [11]. The features of a sound beam as it propagates may indicate nonlinear and damping parameters of the medium and initial perturbations at the exciter. Also, they may be useful in medical and technical applications, where the definition of the focal distance and peak magnitudes of pressure in the focal area is of great importance. The nonlinear focal distance may be larger or shorter that the geometrical focus. The interest of the nonlinear phenomena of sound beams has been growing in the last decades due to applications in non-equilibrium media such as relaxing gases and gases with chemical reactions. Sound is enhanced in these media under certain conditions. In general, the acoustic activity acts as if the viscosity is negative and leads to unusual propagation of sound and relative nonlinear phenomena. Flows with an inflow of external energy also may behave unusually. In particular, self-refraction of the beam reveals unexpected features. This concerns the formation of nonlinear focus and peak magnitude of pressure in the beam. In Newtonian flows, the ”straightening” of a focused-wave front is always observed, but in open flows, the front may bend even more in the course of propagation. The details of self-refraction in some non-equilibrium media may be found in [12, 13].

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