SENSITIVITY-BASED STABILITY EVALUATION OF CLOSED PIPING NETWORKS

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Abstract: Aiming at a closed-loop water system of HVAC engineering, the authors put forward an evaluation method of systems' stability based on sensitivity. It has three evaluating indexes: α – the summation of flow changes in other subcircuits influenced by resistance change in a certain subcircuit; β – the summation of flow changes in a certain subcircuit influenced by resistance changes in other subcircuits; γ – average β (or α) value of each subcircuit, reflecting the strength of regulating interference between subcircuits. The method is used to analyze the stability of a reverse return system (RRS) and a direct return system (DRS). The DRS subcircuit farthest from the heat source and the middle RRS are the least stable. Stability of the whole RRS is inferior to that of the DRS.

Keywords: closed-loop water system, sensitivity, stability, evaluation

1. Foreword

In fluid systems, resistance change in any branch (*i.e.* a line between two arbitrary nodal points) will alter the flow in other branches and itself. Sensitivity is a measuring index that can reflect this relationship. A method based on sensitivity is proposed in this article that evaluates the stability of two basic forms of closed piping networks, *viz.* the direct return system and the reverse return system.

2. Definition of sensitivity

When the resistance of branch i of a fluid network has a change Δq_j which causes a flow change in branch j, Δq_j, and $\Delta s_i \to 0$, we obtain:

$$d_{ij} = \lim_{\Delta s_i \to 0} \frac{\Delta q_j}{\Delta s_i} = \frac{\partial q_j}{\partial s_i}, \tag{1}$$

where d_{ij} is the sensitivity of the flow in branch j to the resistance of branch i.

In a piping network that contains m branches there are $m\times m$ sensitivities, which can be expressed by the following matrix:

$$D=\begin{bmatrix} d_{11} & d_{12} & \dots & d_{1m} \\ d_{21} & d_{22} & \dots & d_{2m} \\ & & \vdots & \\ d_{m1} & d_{m2} & \dots & d_{mm} \end{bmatrix}. \tag{2}$$

In matrix D, elements of row i represent sensitivity of the flow of each branch to the resistance of branch i. Elements of column j represent the sensitivity of the flow in branch j to the resistance of each branch.

3. Calculating sensitivity

There are two ways to calculate sensitivity, as follows.

(1) One way is establishing nodal point flow equilibrium equations and loop pressure equilibrium equations, creating a derivative of each branch's resistance, obtaining the algebraic equations, and the algebraic solutions are the sensitivities of the flow in each branch to the resistance of each branch.

The following is a simple example (see Figure 1) with two subcircuits, to illustrate the method. $E-1-F$ is branch 1, $E-2-F$ is branch 2, $E-p-F$ is branch 3. There are two nodal points, E and F, so we can establish three loop pressure equilibrium equations, but only two of them will be independent. Assuming that the pump's characteristic is $H=f(q_3)$, the piping network equations are as follows:

$$\begin{cases} q_3-q_1-q_2=0, & (3)\\ s_1q_1^2-s_2q_2^2=0, & (4)\\ f(q_3)-s_3q_3^2-s_1q_1^2=0, & (5)\end{cases}$$

where q_i is the flow in branch i and s_i is the resistance of branch i.

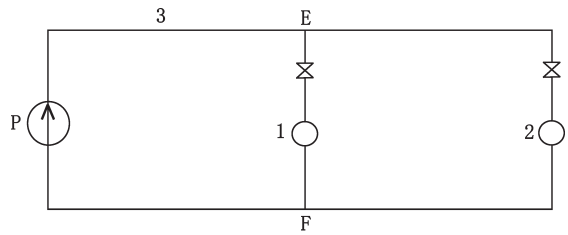

Figure 1. Sketch of a simple piping network

Making the derivative of the above three equations to s_1, s_2, s_3, we obtain:

$$\frac{\partial q_3}{\partial s_i}-\frac{\partial q_1}{\partial s_i}-\frac{\partial q_2}{\partial s_i}=0 \quad i=1,2,3 \tag{6}$$

$$\begin{cases} q_1^2+2s_1q_1\dfrac{\partial q_1}{\partial s_1}-2s_2q_2\dfrac{\partial q_2}{\partial s_1}=0 & (7)\\ 2s_1q_1\dfrac{\partial q_1}{\partial s_2}-\left(q_2^2+2s_2q_2\dfrac{\partial q_2}{\partial s_2}\right)=0 & (8)\\ 2s_1q_1\dfrac{\partial q_1}{\partial s_3}-2s_2q_2\dfrac{\partial q_2}{\partial s_3}=0 & (9)\end{cases}$$

$$\begin{cases} \dfrac{\partial f}{\partial q_3}\dfrac{\partial q_3}{\partial s_1} - 2s_3q_3\dfrac{\partial q_3}{\partial s_1} - \left(q_1^2 + 2s_1q_1\dfrac{\partial q_1}{\partial s_1}\right) = 0 & (10) \\ \dfrac{\partial f}{\partial q_3}\dfrac{\partial q_3}{\partial s_2} - 2s_3q_3\dfrac{\partial q_3}{\partial s_2} - 2s_1q_1\dfrac{\partial q_1}{\partial s_2} = 0 & (11) \\ \dfrac{\partial f}{\partial q_3}\dfrac{\partial q_3}{\partial s_3} - \left(q_3^2 + 2s_3q_3\dfrac{\partial q_3}{\partial s_3}\right) - 2s_1q_1\dfrac{\partial q_1}{\partial s_3} = 0 & (12) \end{cases}$$

There are nine equations, (6)–(12), when the resistance and flow of each branch are given; we can get $\frac{\partial q_j}{\partial s_i}$, $i=1, 2, 3$, $j=1, 2, 3$, namely nine sensitivity values.

(2) The other way is calculating directly according to the definition of sensitivity. The resistance of each branch a piping network is s_1, $s_2,\ldots$, s_m the flow is q_1, $q_2,\ldots$, q_m. With regard to the resistance of branch i, we add an increment Δs_i, solve the piping network equations, and obtain the flow of each branch, q_1', $q_2',\ldots$, q_m', calculating as follows:

$$d_{ij} = \frac{q_j - q_j'}{\Delta s_i}.$$

We reduce the value of Δs_i till the deviation of the two d_{ij} values is in accordance with the precision requirements; this d_{ij} is the sensitivity that of flow in branch j to the resistance of branch i.

4. Stability evaluation of a closed piping network

Obviously, high sensitivity means poor stability and low sensitivity means good stability. A method was proposed in [1] using sensitivity to evaluate systems' stability, which the present authors believe to require the following two improvements when applied to a closed-loop water system in heating and air-conditioning projects:

(a) in stability evaluation of any given branch, the sensitivity of the flow in this branch to the resistances of other branches must be considered, but the sensitivity of the flow in this branch to its own resistance should be neglected;

(b) we are concerned with regulating interference between subcircuits (but not branches) and, in fact, the sum of the flow in each subcircuit is the system's total flow. When resistance in the system is changed, the flow change of each subcircuit can represent the change of the whole system. Thus, we exaggerate the system's reaction if we superpose the flow change of each branch and each subcircuit.

We propose the following method for evaluation of closed water loop systems:

(1) if there are n subcircuits in m branches of a piping network, we separate the $n\times n$ subcircuit sensitivity submatrix from the $m\times m$ sensitivity matrix in order to analyze and evaluate the interference of each subcircuit;

(2) $\alpha_i = \dfrac{1}{n-1}\left(\sum_{j=1}^{i-1} d_{ij} + \sum_{j=i+1}^{n} d_{ij}\right)$, $i=1,2,\ldots n$. α_i is the unit resistance change of branch i that causes the average flow change in other subcircuits. It reflects the influence of the resistance change in branch i on the flow change of other subcircuits, which is called the influence of branch i;

(3) $\beta_j = \dfrac{1}{n-1}\left(\sum_{i=1}^{j-1} d_{ij} + \sum_{i=j+1}^{n} d_{ij}\right), \quad j=1,2,\dots n.$ β_i is the average flow change of subcircuit j caused by changes resistance in other subcircuits. It reflects the total flow change influenced by other subcircuits' resistance change and is called the influenced degree of subcircuit j. Obviously, the greater β_i, the poorer the stability of subcircuit j; the smaller β_i, the better the stability;

(4) $\gamma = \dfrac{1}{n}\sum_{j=1}^{n}\beta_j = \dfrac{1}{n}\sum_{i=1}^{n}\alpha_i$. γ is the average β or α value of each subcircuit, reflecting the strength of regulation interference between subcircuits. When value of γ is great, stability is poor; it is good when the value of γ is small.

5. Exemplary calculation and analysis

5.1. Direct Return System (DRS)

Distribution of resistances in a DRS containing six subcircuits (see Figure 2) is shown in Table 1.

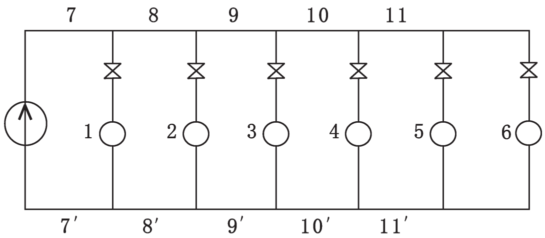

Figure 2. Sketch of a DRS

Table 1. Distribution of resistances in a DRS (h^2/m^5)

Branch	1	2	3	4	5	6	7	8
S	1.67	1.42	1.1	0.74	0.5	0.5	0.005	0.005
Branch	9	10	11	7′	8′	9′	10′	11′
S	0.01	0.02	0.03	0.005	0.005	0.01	0.02	0.03

Note: 1–6 are subcircuits, the others are pipe sections.

With the pump's characteristic of $H = 20.25 - 0.05625Q - 0.003Q^2$, it can be easily shown that if the flow in each subcircuit is $4\,m^3/h$, we obtain the following sensitivity matrix:

$$D = \begin{bmatrix} -1.1097 & 0.0483 & 0.0483 & 0.0483 & 0.0483 & 0.0483 \\ 0.0453 & -1.2725 & 0.0872 & 0.0872 & 0.0872 & 0.0872 \\ 0.0443 & 0.0881 & -1.572 & 0.1876 & 0.1876 & 0.1876 \\ 0.0412 & 0.0923 & 0.1882 & -2.1792 & 0.5171 & 0.5171 \\ 0.0362 & 0.0892 & 0.1830 & 0.4685 & -2.8337 & 1.1186 \\ 0.0362 & 0.0892 & 0.1830 & 0.4685 & 1.1186 & -2.8337 \end{bmatrix}$$

Calculated values of α, β and γ are listed in Table 2.

We can see from the results shown in Table 2 that the best stability is that of subcircuit 1; otherwise, the farthr the subcircuit from the heat source, the poorer its

Table 2. Calculated values of α, β and γ

Branch	1	2	3	4	5	6
α	0.0483	0.0788	0.1390	0.2712	0.3791	0.3791
β	0.0406	0.0794	0.1390	0.2520	0.3918	0.3918
γ	0.2158					

stability. The two terminal subcircuits are parallel, so they have the same stability. Values of α and β have the same order of magnitude for each subcircuit, which indicates that the influence of a given subcircuit on other subcircuits and the influence of other subcircuits on each subcircuit are strongly related.

5.2. Reverse return system (RRS)

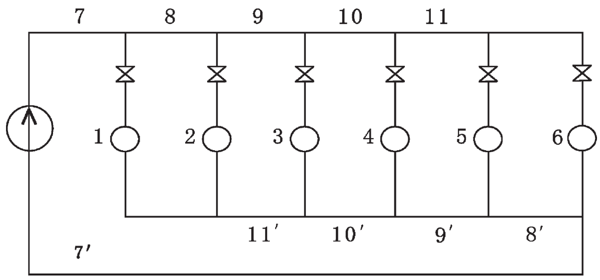

Figure 3. Sketch of a RRS

An RRS with six subcircuits is shown in Figure 3 for comparison with the DRS: the main supply pipe uses the same resistance values as those of the DRS, reversing pipe Section 8′, 9′, 10′, 11′ of the DRS. As the minimum resistance of each subcircuit is the same as in the DRS, $S_1 = S_6 = 0.665$, $S_2 = S_5 = 0.540$, $S_3 = S_4 = 0.500$, $S_7 = 0.217$, the pump's characteristic remains unchanged and the flow in each subcircuit is still $4\mathrm{m}^3/\mathrm{h}$, we obtain the following subcircuit sensitivity matrix:

$$D = \begin{bmatrix} -2.5569 & 0.4186 & 0.2856 & 0.0967 & 0.0511 & 0.0064 \\ 0.4603 & -2.9589 & 0.4360 & 0.1984 & 0.0718 & 0.0430 \\ 0.3623 & 0.5053 & -3.1019 & 0.4752 & 0.2946 & 0.1991 \\ 0.1991 & 0.2946 & 0.4752 & -3.1019 & 0.5053 & 0.3623 \\ 0.0430 & 0.0718 & 0.1984 & 0.4360 & -2.9589 & 0.4603 \\ 0.0064 & 0.0511 & 0.0967 & 0.2856 & 0.4186 & -2.5560 \end{bmatrix}$$

Calculated values of α, β and γ are given in Table 3.

Table 3. Calculated values of α, β and γ

Subcircuit	1	2	3	4	5	6
α	0.1717	0.2419	0.3673	0.3673	0.2419	0.1717
β	0.2142	0.2683	0.2984	0.2984	0.2683	0.2142
γ	0.2603					

We can see from the values of β found in Table 3, that: (1) the stability of the DRS is symmetrical, due to the symmetrical characteristic of sensitivity, and

(2) stability is the best in the two terminal subcircuits, the poorest in the middle two, and these conclusions agree with [2].

5.3. Comparison of the DRS and the RRS

(1) The nearer to the heat source, the better the stability of DRS subcircuits; the farther from the heat source, the poorer the stability. However, with regard to the RRS, the nearer to the two ends, the better the stability; the more to the middle, the poorer the stability.
(2) There are significant differences in stability among the DRS subcircuits, while in the RRS stability is distributed relatively well.
(3) The stability of subcircuits 1, 2, 3 and 4 of the DRS is better than in the RRS, while the stability of subcircuits 5 and 6 of the RRS is better than in the DRS. Thus, overall stability of the DRS is inferior to that of the RRS.

6. Conclusions

Aiming at a closed-loop water system of HVAC engineering, a method of evaluating systems' stability based on sensitivity is put forward in this paper and applied to calculate and analyze the stabiliy of a direct return system (DRS) and a reverse return system (RRS). It has been found that the farther a DRS subcircuit is from the heat source, the poorer its stability: the nearer it is, the better the stability. The stability of the DRS is symmetrical and the best in the two terminal subcircuits, the worst in the middle. The overall stability of the DRS is inferior to that of the RRS.

References

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