APPLICATION OF THE MURNAGHAN MODEL IN ANALYSIS OF NON-LINEAR ELASTIC MATERIAL PROPERTIES OF PVC-COATED FABRIC

ANDRZEJ AMBROZIAK

Department of Structural Mechanics and Bridges Structures, Faculty of Civil and Environmental Engineering, Gdansk University of Technology, Narutowicza 11/12, 80-952 Gdansk, Poland

ambrozan@pg.gda.pl

(Received 22 April 2006)

Abstract: The aim of the present paper is to propose a method of laboratory tests necessary for identification of non-linear elastic properties of the PVC-coated Panama fabric often used for hanging roofs. Two methods of describing the fabric’s non-linear behaviour are investigated: piece-wise linear relations between stress and strain are assumed and the Murnaghan model of solid behaviour is applied. The material parameters are specified on the basis of uniaxial constant strain rate tensile tests in the warp and weft directions. Techniques based on the least squares methods are applied in the identification process.

Keywords: non-linear elastic modelling, Murnaghan model, PVC-coated fabric

1. Introduction

Many theoretical laws have been developed for constitutive modelling of PVC-coated fabrics [1, 2]. Various concepts of modelling can be assumed for warp, weft and coating [3], depending on the type of loading and the working conditions. Polymer structures of a fabric may exhibit viscoelastic [4–6], viscoplastic [7] or characteristics [8–11]. The choice of the model is conditioned by the type of coated fabric and the considered kind of loading. In this paper, the PVC-coated Panama fabric is generally assumed to possess non-linear elastic properties. This type of equations can be successfully used in calculations of form finding analysis, initial pre-stressing of a hanging roof, for dynamic analysis of wind action, etc.

2. Review of constitutive models of coated fabrics

Numerous constitutive models have been recently developed to describe PVC-coated fabrics. The study of coated fabrics dates back to the early work of Hass [12] and Pierce [13]. These authors presented the crimp theory and described the geometrical and mechanical force models of membrane structures made of woven
fabric. The cross-sectional shape of threads and the crimp pattern were discussed in the crimp theory, while the mechanical properties such as stress and strain were omitted for the sake of mathematical simplicity. The trellis model originally described by Weissenberg [14] and developed by Kilby [15] was historically following model applied to describe the deformations of coated fabrics. In the trellis model, the effect of crimp exchange was neglected. Kawabata et al. [16] presented the unit cell model, alternatively referred to as the representative volume element approach. It is one of the methods to obtain material properties for technical materials, knitted or braided fabrics. Wilde [17] studied the problem of dimensioning an unstrained plane membrane to fit the desired surface in the strained state, using the theory of a membrane formed by inextensible cords (cf. Rivlin [18]).

Later, Branicki [19] proposed the dense net model, described in detail in the following section. Then, a model based on the micro-mechanical behaviour of a representative unit cell was developed by Nayfeh and Kress [20]. In this model, the global constitutive relationships were consistently derived from the total system strain energy. Various aspects of constitutive modelling for PVC-coated fabrics and novel experimental testing procedures, as well as rheological parameters for viscoelastic materials, were discussed by Argyris et al. [5]. Tabiei and Jiang [21] developed the micro-mechanical model describing woven fabrics. Karayaka and Kurath [22] applied the micro-mechanical model in composite finite element analysis based on non-linear stress-strain relationships. Kato et al. [23] proposed a formulation of the continuum constitutive equations for fabric membranes. The formulation made use of the fabric lattice model where the structure of fabric membranes was replaced with an equivalent structure composed of truss bars representing the thread net and the coating material. Kuwazuru and Yoshikawa [24] described a pseudo-continuum model for a plain-weave fabric, based on an improved definition of the strain-displacement relationship (see Kuwazuru and Yoshikawa [25] for details). The authors assumed that the fabric’s deformations in the pseudo-continuum model are transformed into axial tensile and transverse compressive strains determined individually for the warp and the weft. Xue et al. [26] presented a non-orthogonal constitutive model to characterize the woven composites. Relationships between the stresses and strains in the global coordinates were established on the basis of stress and strain analysis in the orthogonal and non-orthogonal coordinates and the rigid body rotation matrices.

In the era of the Internet, it may be worth checking out Arcaro’s web page [27], featuring information on design, analysis and interesting references. Arcaro [27] has also developed a simple analytical procedure for cable network structures and a methodology for finding the shape of tension structures.

3. The model

Material composites, including technical woven fabrics, used in membrane roof construction, hanging and pneumatic structures require adequate constitutive modelling describing the material’s complex non-linear behaviour.

We usually have two families of threads in a coated fabric (the warp and the weft), which may change their direction in the deformation process. Therefore, the typical isotropic or orthotropic constitutive equations available in every commercial
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can not be used in its analysis. The plane stress state finite element can be applied to express the fabric’s typical properties with a special substructure capable of describing the behaviour of thread families (especially the changing angle between these families during deformation). Such substructure is included in the dense net model proposed in the paper. The dense net model [19, 28] belongs to a group of continuous models, in which three important assumptions are made:

- Stresses $T_{11}$ and $T_{22}$ in given thread families of the warp and the weft depend solely on strains $\gamma_{11}$ and $\gamma_{22}$ in the same direction:
  
  $$
  T_{11} = F_1(\gamma_{11}) \cdot \gamma_{11} \\
  T_{22} = F_2(\gamma_{22}) \cdot \gamma_{22}
  $$

  (1)

  where $F_1(\gamma_{11})$ and $F_2(\gamma_{22})$ specify the tension stiffness of the warp and weft thread families. These parameters are experimentally determined from uniaxial tension tests in the warp and weft directions, respectively. The following matrix form of the substructure’s constitutive relations can be obtained for the dense net model:

  $$
  T = \begin{bmatrix} T_{11} \\ T_{22} \end{bmatrix} = \begin{bmatrix} F_1(\gamma_{11}) & 0 \\ 0 & F_2(\gamma_{22}) \end{bmatrix} \begin{bmatrix} \gamma_{11} \\ \gamma_{22} \end{bmatrix} = F(\gamma) \cdot \gamma.
  $$

  (2)

- The material of the warp and the weft is isotropic.
- The warp and weft threads can carry stretching forces only.

An important problem is the choice of range and kind of laboratory tests from which the material parameters are identified. Uniaxial tension tests were chosen here, according to the assumptions of the dense net model. An alternative to this approach would be biaxial tests, but they pose a problem in the selection of the ratio of tensile forces in directions of the warp and weft threads. The ratio’s determination in laboratory tests is questionable, because in real structures a different ratio can be observed at each point. There are special constitutive models in which biaxial tensile tests are required to identify the material parameters. For instance, the results of biaxial tests for stress proportions 1:1, 5:1 and 1:5 ($\sigma_{\text{warp}} : \sigma_{\text{weft}}$) are presented in [29] for the fabric model proposed by Day in [30]. Biaxial tensile tests may also be performed for an assumed proportion of strains. For example, biaxial test results for the strain proportion $\varepsilon_{\text{warp}} : \varepsilon_{\text{weft}} = 2$ are given in [31].

In order to derive the elasticity matrix in a local orthogonal system, it is assumed that the warp threads’ family, $\xi_1$, is parallel to the $x_1$ axis of the local Cartesian coordinate system in the plane stress state finite element and that the weft family, $\xi_2$, is inclined at angle $\alpha$ (see Figure 1).

Figure 1. Stress components in the plane stress state
The relation between the plane stress state components, \( \sigma \), in local coordinates \( x(x_1, x_2) \) of the finite element and the substructure forces in the thread families, \( T \) (specific forces), in directions \( \xi(\xi_1, \xi_2) \) is defined by transformation matrix \( C \):

\[
\sigma = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 1 & \cos^2 \alpha \\ 0 & \sin^2 \alpha \\ 0 & \sin \alpha \cos \alpha \end{bmatrix} \cdot \begin{bmatrix} T_{11} \\ T_{22} \end{bmatrix} = C \cdot T. \tag{3}
\]

Additionally, the relation between strains \( \gamma_{11} \) and \( \gamma_{22} \) and strains \( \varepsilon_{11}, \varepsilon_{22} \) and \( \varepsilon_{12} \) in local coordinates \( x \), can be specified as follows:

\[
\gamma = \begin{bmatrix} \gamma_{11} \\ \gamma_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \cos^2 \alpha & \sin^2 \alpha & \sin \alpha \cos \alpha \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2 \varepsilon_{12} \end{bmatrix} = C^T \cdot \varepsilon. \tag{4}
\]

According to Equations (2)–(4), the relation between plane stress state components \( \sigma \) and strains \( \varepsilon \), can be specified as:

\[
\sigma = C \cdot T = C \cdot F \cdot \gamma = \underline{C} \cdot F \cdot C^T \cdot \varepsilon = D \cdot \varepsilon, \tag{5}
\]

where \( D \) defines the elasticity matrix and is expressed as follows:

\[
D = \begin{bmatrix} F_1(\gamma_{11}) + F_2(\gamma_{22}) \cdot \cos^4 \alpha & F_2(\gamma_{22}) \cdot \sin^2 \alpha \cdot \cos^2 \alpha & F_2(\gamma_{22}) \cdot \sin \alpha \cdot \cos^3 \alpha \\ F_2(\gamma_{22}) \cdot \sin \alpha \cdot \cos^2 \alpha & F_2(\gamma_{22}) \cdot \sin^4 \alpha & F_2(\gamma_{22}) \cdot \sin^3 \alpha \cdot \cos \alpha \\ F_2(\gamma_{22}) \cdot \sin^3 \alpha \cdot \cos \alpha & F_2(\gamma_{22}) \cdot \sin^2 \alpha \cdot \cos^2 \alpha & F_2(\gamma_{22}) \cdot \sin \alpha \cdot \cos^3 \alpha \end{bmatrix}. \tag{6}
\]

The dense net model is capable of expressing the most important properties of fabric materials, which can be determined in uniaxial tension tests in the warp and weft directions only. The model includes the change of the angle between thread families of the warp and the weft during the fabric’s deformation. The angle is calculated on the basis of current values of stress components \( \sigma_{22} \) and \( \tau_{12} \) in the plane stress state from the following relation:

\[
\alpha = \arctan \frac{\sigma_{22}}{\tau_{12}}. \tag{7}
\]

For a typical coated fabric, the initial value of the angle between thread families is usually \( \alpha = \alpha_0 = 90^\circ \).

### 4. Laboratory tests

The non-linear elastic properties of the PVC-coated Panama fabric were determined on the basis of laboratory tests [32]. The fabric was manufactured from polyester threads of P2/2 weave coated with PVC layers on both sides. A Zwick 147670 computer-operated strength-testing machine was used in the tests (see Figure 2). Specimens 0.05m wide and 0.20m long were cut out in the warp and weft directions. Other important parameters, including the weight of 870g/m² and the initial weave angle of \( \alpha_0 = 90^\circ \) were specified by the manufacturer. The results of uniaxial tension tests in the warp and weft threads’ directions with constant strain rates of \( 5 \times 10^{-3} \text{s}^{-1} \), \( 10^{-3} \text{s}^{-1} \) and \( 10^{-4} \text{s}^{-1} \) are given in Figures 3–5 [32]. Various strain rates were used to ascertain the range in which the identified values could be used. Non-linear elastic properties were determined in the 0.0–0.11 strain range for the
warp and the 0.0–0.16 range for the weft. In these strain ranges, the rheological effect can be neglected (see [33] for details).

5. Piecewise linear elastic constitutive description

As shown in Figures 3–5, there are some intervals in which a linear relation between strains and stresses can be assumed; therefore, a piecewise linear description has been proposed. Three coefficients of a straight line for the weft, \( F_2A, F_2B, F_2C \), and the warp, \( F_1A, F_1B, F_1C \) (see Figure 8), specify the longitudinal moduli. The elastic linear model, described below, was used in the admissible strains’ range.

\[ \begin{align*}
\text{Figure 2.} & \quad \text{The gripping jaws of the testing machine} \\
\text{Figure 3.} & \quad \text{Test results: the strain-stress curve for } \dot{\varepsilon} = 5 \cdot 10^{-3} \text{s}^{-1}
\end{align*} \]
Figure 4. Test results: the strain-stress curve for $\dot{\varepsilon} = 10^{-3} \text{s}^{-1}$

Figure 5. Test results: the strain-stress curve for $\dot{\varepsilon} = 10^{-4} \text{s}^{-1}$

5.1. The model

The relation between stress, $S$, and strain, $E$, tensors in the linear elastic model has the following form (see [34–36] for details):

$$S_{ij} = C_{ijkm} \varepsilon_{km}, \quad S = C \otimes E.$$  \hspace{1cm} (8)

The potential energy of a material’s adiabatic deformation can be given as:

$$\Phi = \frac{1}{2} S \otimes E.$$  \hspace{1cm} (9)

Consequently, the stresses and the elasticity tensor are as follows:

$$S_{ij} = \frac{\partial \Phi}{\partial \varepsilon_{ij}} = \frac{\partial \Phi}{\partial U_{ij}}, \quad S = \frac{\partial \Phi}{\partial E}.$$  \hspace{1cm} (10)

$$C_{ijkm} = \frac{\partial^2 \Phi}{\partial \varepsilon_{ij} \partial \varepsilon_{km}}, \quad C = \frac{\partial S}{\partial E} = \frac{\partial}{\partial E} \left( \frac{\partial \Phi}{\partial E} \right).$$  \hspace{1cm} (11)
The potential energy accumulated in an isotropic elastic solid can be expressed as:

\[ \Phi = \frac{1}{2} (\lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}) \varepsilon_{ij} = \frac{1}{2} \lambda (\varepsilon_{mm})^2 + \mu (\varepsilon_{ij})^2. \]  

(12)

Therefore, Hooke’s law for an isotropic material will be:

\[ S_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}, \quad S = \lambda \text{tr}(E) I + 2\mu E, \]  

(13)

where \( \lambda, \mu \) are the Lamé constants.

5.2. The parameters

In order to describe the uniaxial stress state, we can simplify the energy and stress notations, as strain \( \varepsilon_1 \) is the principal strain and \( \varepsilon_3 = \varepsilon_2 \). According to Equations (12) and (13), the components of the stress tensor can be calculated from the following equations:

\[ \Phi = \frac{1}{2} \lambda (\varepsilon_{mm})^2 + \mu (\varepsilon_{ij})^2 \Rightarrow S_{ij} = \frac{\partial \Phi}{\partial \varepsilon_{ij}} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}, \]

(14)

\[ S_{11} = \lambda (\varepsilon_1 + 2\varepsilon_2) + 2\mu \varepsilon_1 \quad \text{and} \quad S_{22} = S_{33} = \lambda (\varepsilon_1 + 2\varepsilon_2) + 2\mu \varepsilon_2. \]

It should be noted that stress component \( S_{11} \) is a function of the unknown strain \( \varepsilon_2 \). This strain can be determined from the assumption of the uniaxial stress state, \( S_{22} = S_{33} = 0 \). Subject to this assumption, it is possible to calculate \( \varepsilon_2 \) directly:

\[ \lambda (\varepsilon_1 + 2\varepsilon_2) + 2\mu \varepsilon_2 = 0 \Rightarrow \varepsilon_2 = -\frac{\lambda \varepsilon_1}{2\lambda + 2\mu}. \]

(15)

Lamé constants of \( \lambda = 4989616745 \text{N/m}, \mu = 318642 \text{N/m} \) can be assumed on the basis of laboratory tests (e.g. warp_31, see Figure 6). The longitudinal elasticity modulus, \( F \), can be determined from known values of the Lamé constants, as follows:

\[ F = \frac{\mu (3\lambda + 2\mu)}{\lambda + \mu} = \frac{318642(3 \cdot 4989616745 + 2 \cdot 318642)}{4989616745 + 318642} = 955907 \text{N/m} \]

(16)

It is also possible to obtain the longitudinal modulus, \( F \), directly from laboratory tests:

\[ S_{11} = F \cdot \varepsilon_1. \]

(17)

The value of \( F = 955907 \text{N/m} \) can be obtained from a linear approximation (e.g. warp_31, see Figure 7). This example of elastic parameter determination has confirmed the assumed identification procedure and will be extended in the following section to the Murnaghan model.

The final results of are given in Table 1 and Figure 8.

Table 1. Non-linear elastic properties of coated fabric Panama [33]

<table>
<thead>
<tr>
<th></th>
<th>[kN/m]</th>
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<td></td>
</tr>
<tr>
<td>F1A</td>
<td>904</td>
<td>( \varepsilon_{P1} = 0.0119 )</td>
</tr>
<tr>
<td>F1B</td>
<td>176</td>
<td>( \varepsilon_{P2} = 0.093 )</td>
</tr>
<tr>
<td>F1C</td>
<td>471</td>
<td>( \varepsilon_{P3} = 0.110 )</td>
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<tr>
<td>weft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2A</td>
<td>187</td>
<td>( \varepsilon_{P1} = 0.039 )</td>
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<tr>
<td>F2B</td>
<td>146</td>
<td>( \varepsilon_{P2} = 0.1495 )</td>
</tr>
<tr>
<td>F2C</td>
<td>340</td>
<td>( \varepsilon_{P3} = 0.160 )</td>
</tr>
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</table>
6. The Murnaghan model-based description of threads’ behaviour

6.1. The model

The material model of the Murnaghan solid has been chosen for this analysis. In the Murnaghan solid model, potential energy is given by the following equation (cf. Murnaghan [37], Guz [38]):

\[
\Phi = \frac{1}{2} \lambda (E_{mm})^2 + \mu (E_{ij})^2 + \frac{1}{3} AE_{ij}E_{im}E_{jm} + B (E_{ij})^2 E_{mm} + \frac{1}{3} C (E_{mm})^3,
\]

(18)

where \( \lambda, \mu \) are the Lamé constants and \( A, B, C \) are Murnaghan constants, while \( E_{ij} \) are components of the Lagrange-Green strain tensor. It should be noted that the Murnaghan proposal of energy approximation is not the only one found in the
Figure 8. Graphical interpretation of the $F$ parameters

The invariants of the Lagrange-Green strain tensor can be specified from the principal strain components as follows:

$I_E = \text{tr} (E) = \epsilon_1 + \epsilon_2 + \epsilon_3,$

$II_E = \frac{1}{2} \left( \text{tr} (E^2) - (\text{tr} (E))^2 \right) = \epsilon_1 \epsilon_2 + \epsilon_1 \epsilon_3 + \epsilon_2 \epsilon_3,$

$III_E = \text{det} (E) = \epsilon_1 \epsilon_2 \epsilon_3.$

(19)

With the Lagrange-Green strain tensor invariants Equation (19), the potential energy can be written in the following form (see [40]):

$\Phi = \lambda + 2\mu \left( I_E \right)^2 - 2\mu II_E + \frac{l + 2m}{3} \left( I_E \right)^3 - 2m \left( I_E \right) II_E + nIII_E,$

(20)

where $\lambda$, $\mu$ are the Lamé constants (the same as in the piecewise model) and $l$, $m$, $n$ are the Murnaghan constants. The second Piola-Kirchhoff stress $S$ is derived as the gradient of energy $\Phi$ with respect to $E$:

$S = \left( \frac{\lambda + 2\mu}{3} I_E - (2m - n) II_E + (l + 2m) \right) I + (2\mu + (2m - n) I_E) E + nE^2.$

(21)

In a particular case of the uniaxial stress state, strains $\epsilon_2$ and $\epsilon_3$ are equal. The Lagrange-Green strain tensor invariants have the following form:

$I_E = \text{tr} (E) = \epsilon_1 + 2\epsilon_2,$

$II_E = \frac{1}{2} \left( \text{tr} (E^2) - (\text{tr} (E))^2 \right) = 2\epsilon_1 \epsilon_2 + \epsilon_2^2,$

$III_E = \text{det} (E) = \epsilon_1 \epsilon_2^2.$

(22)

Then, the potential energy can be determined from the following equation:

$\Phi = \frac{\lambda + 2\mu}{2} \left( \epsilon_1 + 2\epsilon_2 \right)^2 - 2\mu \left( 2\epsilon_1 \epsilon_2 + \epsilon_2^2 \right) + \frac{l + 2m}{3} \left( \epsilon_1 + 2\epsilon_2 \right)^3 - 2m \left( 2\epsilon_1^2 \epsilon_2 + \epsilon_1 \epsilon_2^2 + 2\epsilon_2^3 \right) + n \left( \epsilon_1 \epsilon_2^2 \right).$

(23)
Finally, the stress components are:

\[
S_{11} = \frac{\partial \Phi}{\partial \epsilon_1} = (\lambda + 2\mu)(2\epsilon_1 + 4\epsilon_2) - 2\mu \epsilon_2 + (l + 2m)(\epsilon_1 + 2\epsilon_2)^2 - 2m(4\epsilon_1 \epsilon_2 + 5\epsilon_2^2) + n\epsilon_2^2 - 2\mu^2 \epsilon_2 \tag{24}
\]

\[
= \epsilon_1^2(l + 2m) + \epsilon_1(4\epsilon_2l + \lambda + 2\mu) + \epsilon_2(2l - 2m + n + 2\lambda) + \epsilon_1^2 \lambda + \epsilon_1(4\epsilon_2l + \lambda + 2\mu) + \epsilon_2(2l - 2m + n + 2\lambda)
\]

and

\[
S_{22} = S_{33} = \frac{\partial \Phi}{\partial \epsilon_2} = (\lambda + 2\mu)(4\epsilon_1 + 8\epsilon_2) - 2\mu(2\epsilon_1 + 2\epsilon_2) + 2(l + 2m)
\]

\[
= (\epsilon_1 + 2\epsilon_2)^2 - 2m(2\epsilon_1^2 + 2\epsilon_1 \epsilon_2 + 8\epsilon_1 \epsilon_2 + 6\epsilon_2^2) + 2n\epsilon_1 \epsilon_2 = 2(\epsilon_1^2l + \epsilon_1(4l - 2m + n) + \lambda) + 2\epsilon_2^2(2l - m + 2\epsilon_2(\lambda + \mu)) \tag{25}
\]

As in the previous section, it is necessary to solve the \( S_{22} = 0 \) equation, from which the unknown component of strain \( \epsilon_2 \) can be determined, but the solution is complex, as Equation (25) is a second-order polynomial function of \( \epsilon_2 \).

### 6.2. The parameters

The least squares’ regression was applied to determine the material parameters for the Murnaghan model. The Marquardt-Levenberg algorithm [41] was used to find the coefficients of the independent variables that offer the best fit between the equation and the experimental data. On the basis of this numerical algorithm, coefficients \( \lambda, \mu, l, m, n \) were specified for the warp (see Table 2) and the weft (see Table 3) for each laboratory test. The final results of material parameters’ identification (mean values of coefficients \( \lambda, \mu, l, m, n \)) are given in Table 4.

#### Table 2. Murnaghan coefficients – warp

<table>
<thead>
<tr>
<th>( \dot{\epsilon} ) [s(^{-1})]</th>
<th>Test name</th>
<th>( \lambda ) [N/m]</th>
<th>( \mu ) [N/m]</th>
<th>( l ) [N/m]</th>
<th>( m ) [N/m]</th>
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#### 6.3. Numerical simulation

In order to verify the obtained results, numerical simulations of uniaxial tension tests were performed for the established mean values of the Murnaghan coefficients. The results of these calculations are shown in Figures 9–11. Good agreement of the stress-strain relations obtained from numerical simulations and laboratory tests has been observed.
Table 3. Murnaghan coefficients – weft

<table>
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<th>$\mu$ [N/m]</th>
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Table 4. Murnaghan coefficients for PVC-coated Panama fabric

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Figure 9. Calculation results: comparison with laboratory tests ($\dot{\varepsilon} = 5 \cdot 10^{-3}$s⁻¹)

7. Conclusions

The non-linear elastic Murnaghan model has been successfully applied on the basis of uniaxial tension tests to describe the behaviour of a PVC-coated Panama fabric. The model can be applied in a wide range of engineering problems. The material
parameters determined above can be used directly in FEM to analyze structures made of PVC-coated fabrics, where geometric non-linearity is accompanied by the physical non-linearity of the fabric’s material.

References

Application of the Murnaghan Model in Analysis of Non-Linear Elastic Material

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